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Numerical Modelling of Flow Over A Rigid Wavy Surface by LES

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Abstract

A flow field over a rigid wavy surface is numerically generated. The general concept of the large eddy simulation that captures the large scale flow structure is utilized. A coordinate transformation scheme is developed to transform any wavy surface to a flat surface. The numerical computations are carried out in the $\omega - \psi$ domain. The subgrid scales are resolved by utilizing a Smagorinsky subgrid model. The fourth-order central difference and the third-order upwinding schemes are used for the diffusive and convective terms, respectively. Adams-Bashforth and Dufort-Frankel schemes are used to solve the equations of the fluid motion. The computational scheme ensured a non-penetration and a no-slip boundary condition. The velocity profile, and the separation and reattachment locations are found to agree well with an experimental investigation.

1. INTRODUCTION

Applications of flow over wavy surfaces span many areas of interests that include generation of water waves, development and migration of sand dunes in deserts and sediment dunes in rivers. In this study, to improve our understanding of the flow over fully developed sea states, the simulation of wind flow over a rigid wavy surface is conducted. The compliant nature of offshore drilling platforms, being developed for deep water drilling, has increased their sensitivity to the dynamic effects of fluctuations in the wind loads. Limited full-scale information concerning the wind field characteristics over the ocean has prompted the extension of onshore practice for the wind field analysis to offshore practice. However, there exist major differences that concern the variable nature of the sea surface which translates and deforms. Locally, the wind profile may be influenced by the changes in the sea surface and may also influence the turbulence structure. Many previous studies have addressed the topic of wind-wave interaction, but the focus has been in the interaction, wave momentum flux and pressure on the wave surface. Most of the theoretical investigation of wind-wave interaction are based on work by Miles (1957 and 1959), and Benjamin (1959). Most recent studies include modelling of wave boundary layer, based on the nonlinear Reynolds equations in a curvilinear system of coordinates (Chalikov and Makin, 1991). These studies are primarily focused on developing wave boundary layer models for input to wave prediction models.

The numerical simulation of flow over wavy surfaces has been accomplished earlier utilizing Reynolds averaging in conjunction with eddy viscosity and mixing length modelling of turbulence (McLean, 1983; Sengupta and Lekoudis, 1985; Patel, et al., 1991). Britter, et al., (1981) have examined airflow over a two-dimensional hill from analytical considerations. Experimental studies are reported by Hsu et. al., 1981. The present study involves simulation of wind velocity profiles and other flow characteristics over fully developed waves. The first phase, which is reported here, concerns simulation of wind flow over rigid wavy surfaces. This model will be expanded subsequently to include propagating waves.

Among the current simulation methods, the Reynolds Averaged Navier-Stokes (RANS) method and Large Eddy Simulation (LES) method are most popular (e.g., Krettenauer and Schumann, 1992, Liu & Kareem, 1992. Ferziger, 1990; and Murakami, et. al., 1989). Both approaches involve approximating: i) averages of the nonlinear terms, i.e., turbulence model; ii) domain discretization and; iii) solution of the discretized equation. LES is less sensitive to errors inherent in modelling as compared to RANS, hence, the quality of results is less dependent on modelling accuracy. Although, this advantage is not without the penalty concerning additional computational effort.

2. GOVERNING EQUATIONS

The governing equations of motion for wind flow over two-dimensional waves are given by the following averaged Navier-Stokes equations (space-average)

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}(2\nu S_{xx} - R_{xx}) + \frac{\partial}{\partial y}(2\nu S_{xy} - R_{xy}) \quad (1)$$

$$\frac{\partial \bar{v}}{\partial t} + \bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{\partial}{\partial x}(2\nu S_{xy} - R_{xy}) + \frac{\partial}{\partial y}(2\nu S_{yy} - R_{yy}) \quad (2)$$

where $\bar{u}$ and $\bar{v}$ are the mean air velocities in x and y directions ν and ρ are the air kinematic viscosity and air density, respectively. $S_{ij} = 1/2\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$, is the strain rate, P is the mean pressure, and its fluctuation is neglected. The Reynolds stress, $R_{ij} = \overline{u'_i u'_j}$, is assumed to be related to the mean strain rate S_{ij} by

$$R_{ij} = -2\nu_{SGS} S_{ij} + \frac{2}{3} q^2 \delta_{ij} \quad (3)$$

where $i, j = x, y$ directions, q and δ_{ij} are the turbulent intensity and the Knonecker delta. ν_{SGS} is the eddy-viscosity which normally depends on the Reynolds number and the mean strain rate (e.g., Aldama, 1990). Smagorinsky (1963) proposed the following eddy-viscosity expression with only one empirical constant that now is known by his name:

$$\nu_{SGS} = (C\Delta)^2 (2S_{ij}S_{ij})^{1/2} \quad (4)$$

The subgrid coefficients can be adjusted between 0.1 and 0.2 depending on the boundary geometry, flow field and the Reynolds number, and Δ is the mean grid spacing.

The corresponding governing equations in the $\omega - \psi$ formulation are given by

$$\frac{\partial\omega}{\partial t} = \frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y} + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\nu_T\omega)$$
$$-2\left(\frac{\partial^2\psi}{\partial x^2}\frac{\partial^2\nu_T}{\partial y^2} - 2\frac{\partial^2\psi}{\partial x\partial y}\frac{\partial^2\nu_T}{\partial x\partial y} + \frac{\partial^2\psi}{\partial y^2}\frac{\partial^2\nu_T}{\partial x^2}\right) \tag{5}$$

$$\omega = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} \tag{6}$$

in which $\nu_T = \nu + \nu_{SGS}$ is the total kinematic viscosity.

3. COORDINATE TRANSFORMATION

There are several possible schemes to convert the wavy surface to a flat surface. However, for computational convenience a transformation scheme is chosen such that Poisson equation in the transformed coordinates (namely curvilinear coordinates) retains its general form. The coordinate transformation is derived here for application to any boundary surface, that is continuous and two-dimensional.

We will use x and y here to represent the Cartesian coordinates and n and s for curvilinear coordinates. On the curvilinear coordinate system, the boundary surface $y = f(x)$ is flat, i.e., $n = 0$. The relationship between the two coordinates is given by

$$x = x(s, n) \tag{7}$$

$$y = y(s, n)\,. \tag{8}$$

Assuming the transformation Jacobian

$$J = \frac{\partial x}{\partial s}\frac{\partial y}{\partial n} - \frac{\partial x}{\partial n}\frac{\partial y}{\partial s}, \tag{9}$$

is not zero, the first-order and the second-order derivative operators can be achieved and are thus applied to the Poisson equation (6). To guarantee that the Possion equation retains its form, the transformation equations are given by

$$\frac{\partial y}{\partial n} = \frac{\partial x}{\partial s} \tag{10}$$

$$\frac{\partial y}{\partial s} = -\frac{\partial x}{\partial n}. \tag{11}$$

Accordingly, the orthogonal coordinates x and y are transformed to the orthogonal coordinates s and n, based on the above grid transformation equations. For brevity, the details are omitted here.

The relation between $x-y$ and $s-n$ are solved by equations (10) and (11), given the following boundary conditions: boundary condition on $n = 0$, periodical boundary condition on the side walls of the flow field and the free-stream boundary condition, i.e., $\partial y/\partial n = 1.0$, on the top of the flow field. For example, to get an expression for y, the following Poisson equation

$$\frac{\partial^2 y}{\partial s^2} + \frac{\partial^2 y}{\partial n^2} = 0, \tag{12}$$

is solved together with the boundary conditions

$$\frac{\partial y}{\partial s} = \frac{f'(x)}{\sqrt{1+f'^2(x)}} \tag{13}$$

$$\frac{\partial y}{\partial n} = \frac{1}{\sqrt{1+f'^2(x)}}, \tag{14}$$

at $n = 0$. The above boundary condition is based on the assumption that s is the length of the boundary surface from $x = 0$ to x, i.e.,

$$s = \int_0^x \sqrt{1+f'^2(\xi)}\, d\xi, \tag{15}$$

because the boundary is a two-dimension surface. Therefore, the preceding equations in the curvilinear coordinates s and n (parallel and normal to the wavy surface, respectively) are given by

$$\begin{aligned} \frac{\partial \omega}{\partial t} &= \frac{1}{J}\frac{\partial \psi}{\partial n}\frac{\partial \omega}{\partial s} - \frac{1}{J}\frac{\partial \psi}{\partial s}\frac{\partial \omega}{\partial n} + \frac{1}{J}\left(\frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial n^2}\right)(\nu_T \omega) \\ &\quad -2\left(\frac{\partial^2 \psi}{\partial x^2}\frac{\partial^2 \nu_T}{\partial y^2} - 2\frac{\partial^2 \psi}{\partial x \partial y}\frac{\partial^2 \nu_T}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y^2}\frac{\partial^2 \nu_T}{\partial x^2}\right) \end{aligned} \tag{16}$$

$$J\omega = \left(\frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial n^2}\right)\psi \tag{17}$$

where J is the transformation Jacobian.

4. COMMENTS ON NUMERICAL SIMULATION

The diffusive and advective terms in the preceding equations are evaluated using fourth-order finite difference and third-order upwinding schemes, respectively, by taking into consideration the accuracy and the feedback sensitivity (Leonard, 1981). The overall computational scheme involves Adams-Bashforth method, combined with DuFort-Frankel method for the diffusive term (Pinelli and Benocci, 1989; Roche, 1976). The time step is determined by the advective and diffusive Courant-Friedrich-Levy number. The computation is basically conducted over a sinusoidal wave under the periodical boundary condition. At the rigid wavy boundary both non-penetration and no-slip conditions are satisfied following the work by Israeli (1970). The initial condition is set to be the state of zero vorticity superimposed by a small perturbation. It's reported that the perturbation is required to sustain the steady airflow at a proper energy level (Pinelli and Benocci, 1989).

5. RESULTS AND DISCUSSION

Airflow at different Reynolds numbers are investigated by numerical simulation over a periodic, stationary wave. Different wave amplitudes and wavelengths are investigated. Here, as an example, the coordinate transformation is done for a sinusoidal wave $y = A \ \cos(2\pi x/\lambda)$, where the wave amplitude, A, is 1.0 m and the wavelength, λ, is 10.0 m. The curvilinear coordinate $s-n$ mesh 65×65 is constructed over one sinusoidal wavy surface. The mesh was denser in the flow field close to the wave crest and coarser in the flow field close to the wave trough (Fig. 1).

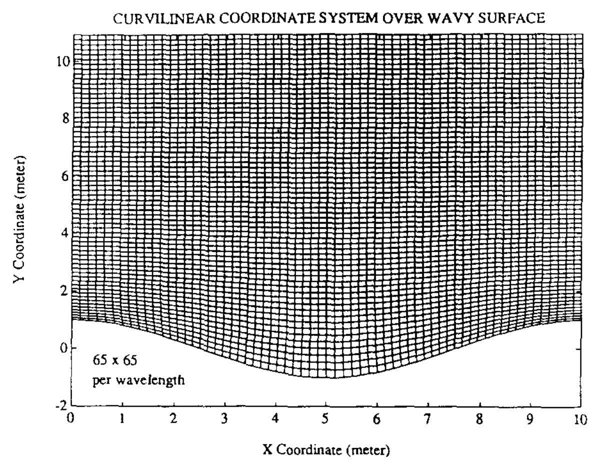

Fig. 1.A curvilinear mesh over a sinusoidal wavy surface.

Simulations of the airflow over the sinusoidal surface are conducted on the $s-n$ mesh described above at $Re = 3,000-300,000$. The free-stream velocity is $u = 45 \ \ cm/sec$ and the time step is equal to 0.001 λ/u. The time sequence plots of the flow parameters show that the separation is originally noted on the leeward side of the wave crest at the 1000th time step, it moves to the wave trough at the 1500th time step, and subsequently moves to the windward of the wave crest and shrinks there at the 2000th time step. The separation moves back to the wave trough at the 2500th time step, and later continues to shift back to the leeward side of the wave crest at the 2800th time step, and stays in the leeward of the wave crest and strengthens itself at the 3000th time step (Fig. 2). Correspondingly, the movement of the vorticity core is observed (Fig. 3). In this study it is observed that the evolution time for a mature separation on the leeward side of the wave crest takes about $3\lambda/u$.

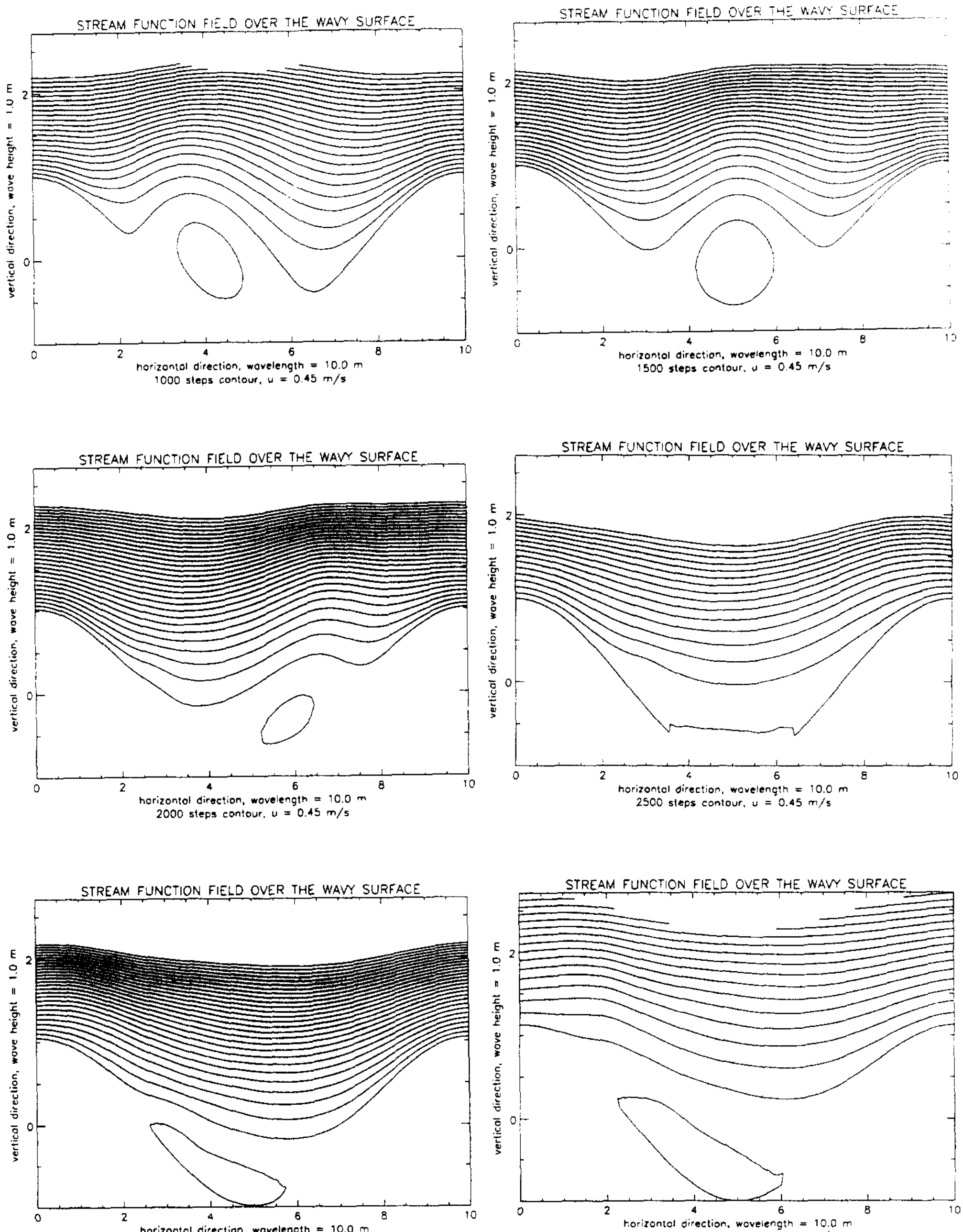

Fig. 2. Streamlines over a sinusoidal wavy surface at different time steps.

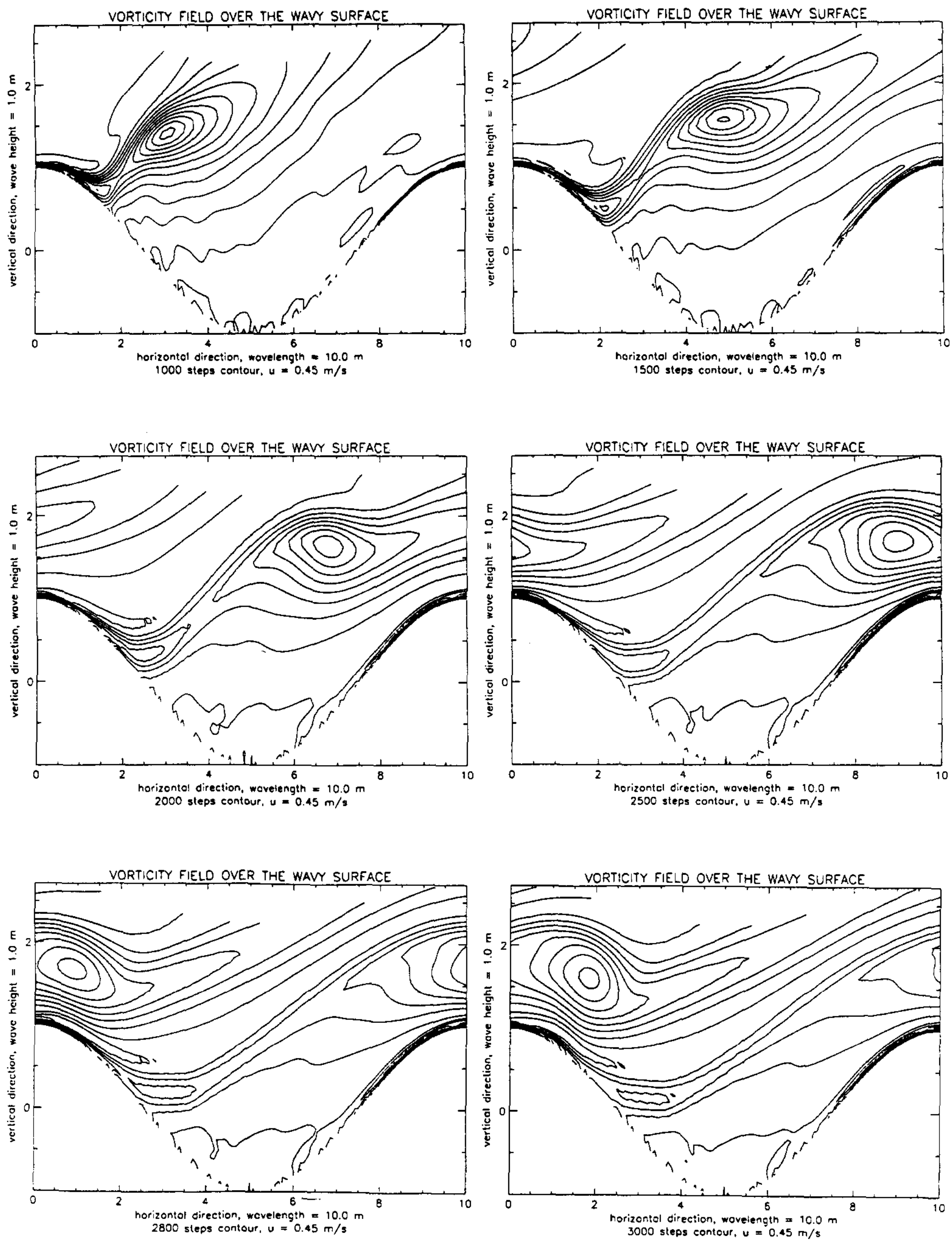

Fig. 3. Vorticity contours of airflow over a sinusoidal boundary at different time steps.

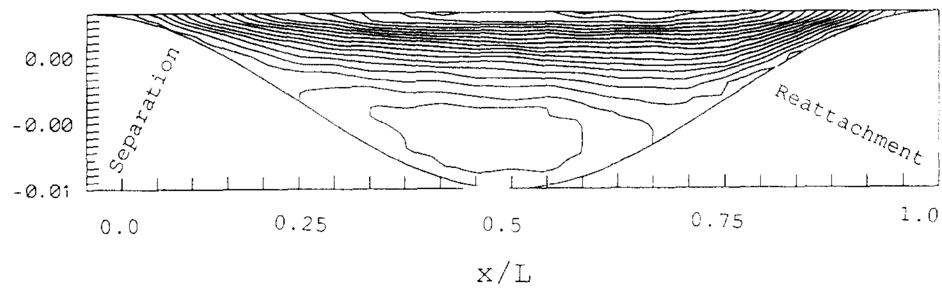

Fig. 4. Streamlines of flow over wavy surface.

To check the validity of our numerical model, another run is made for the air flow $u = 51\ cm/sec$ over the stationary wavy surface, $y = A\ \cos\ (2\pi x/\lambda)$, where $A = 5.08\ mm$ and $\lambda = 50.8\ mm$, and results from our simulation are compared with the experiment conducted by Buckles, et. al. (1989). The contour of the stream function in Figure 4 shows that the separation is at $x/\lambda = 0.1$ and the reattachment is at $x/\lambda = 0.8$, which agrees well with the observed values reported by Buckles, et al., in a water tunnel. Comparisons of the horizontal velocity profiles between our model and the experiment by Buckles, et. al. at different horizontal locations are shown in Figure 5. The results indicate that the simulation is in good agreement with experimental results. This work is being continued and the fluctuating flow field characteristics are being evaluated. The next phase of this study would involve 3-D simulation of flow field over waves.

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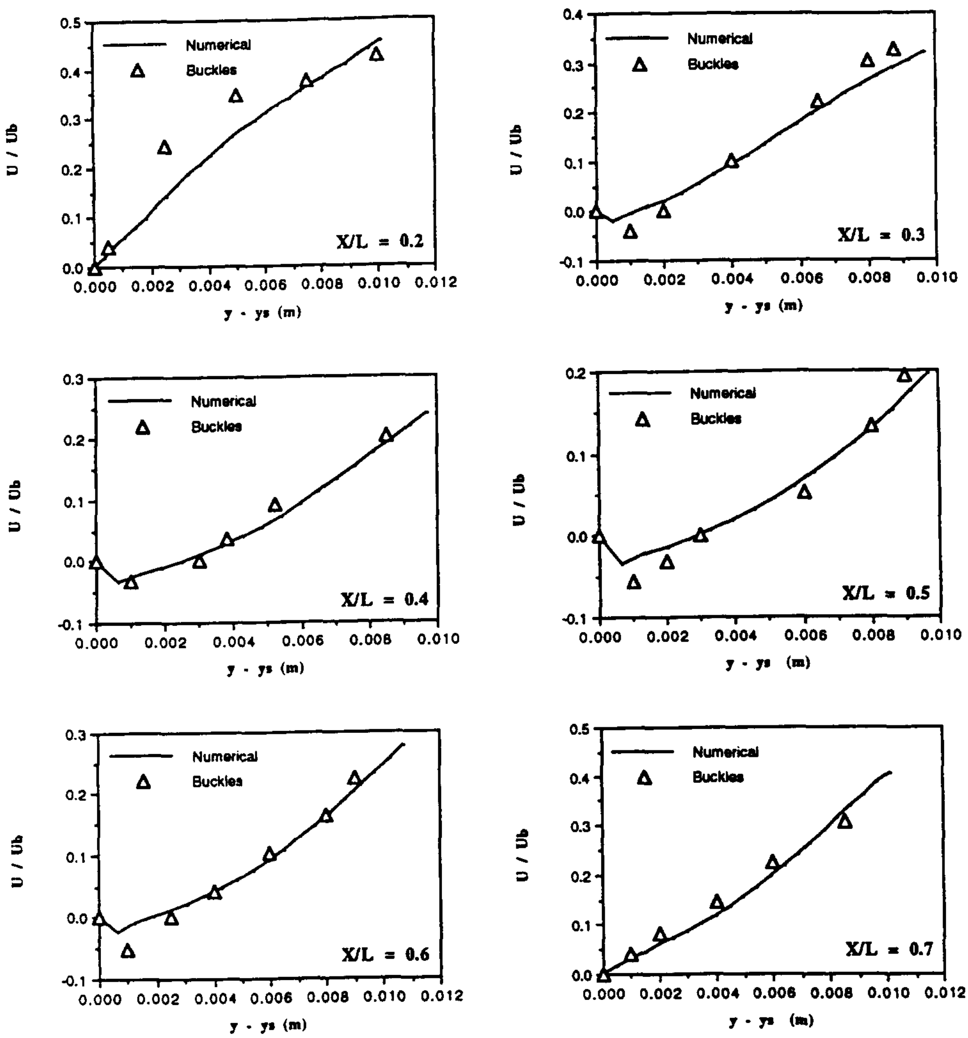

Fig. 5. Comparison of horizontal velocity profiles obtained from numerical and water tunnel experiments.

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