Conditional Simulation of a Gust-Front Wind Field

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ABSTRACT: Measurements in wind engineering are often influenced by the limited number of sensors or difficulty of monitoring at inaccessible locations that impacts the collection of desired information. This gap in information can be filled through the simulation of missing information conditional upon the measured records. This study utilizes a time-frequency framework with a focus on the conditional simulation of non-stationary/non-Gaussian random processes. In this context, an effective tool for stochastic interpolation of non-stationary stochastic processes is presented with application to non-stationary gust-front wind field. The proposed method allows characterization of the non-stationary stochastic processes in terms of time-frequency dependent spatial correlation functions and facilitates the conditional simulation of non-stationary space-time random fields with evolving correlation structure known a priori. Numerical examples concerning the conditional simulation of gust-front wind velocities are presented to generate time series at locations in between measurements consistent with the derived information about the evolutionary characteristics of temporal fluctuations in velocity.

KEYWORDS: Conditional Simulation; Kriging; Hermite Transform; Fourier, Hilbert and Wavelet Transforms; Downburst; Thunderstorm.

1 INTRODUCTION

In the analysis of the wind load effects on structures, the simulation of wind velocity conditional upon the availability of measured records is often required due to the limited number of instruments or difficulty in monitoring at inaccessible locations. Generally, the conditional simulation can be performed utilizing Kriging method or the Conditional Probability Density Function (CPDF) method. The Kriging method was developed by Krige [1] in solving the ore evaluation problem. Vanmarcke *et al.* [2] first applied the Kriging method to conditional simulation problems in earthquake engineering**.** Hoshiya and Maruyama [3] modified the conditional simulation method based on Kriging method by taking into account the corresponding error covariance matrix. Utilizing the orthogonality property between the best estimator and the corresponding error, the modification represents a significant improvement that has made the Kriging method theoretically much cleaner and computationally more efficient. The CPDF method was proposed by Kameda and Morikawa [4] to solve the conditional simulation problems involving earthquake related stochastic processes. In this probabilistic framework, a closed form solution for joint probability density functions for Fourier coefficients is derived through cross-spectral density functions based on Gaussian assumption. A recent study by Shinozuka and Zhang [5] has concluded that the two methods are equivalent when the underlying stochastic process is a one-dimensional uni-variate stationary Gaussian process with zero-mean. Both methods are valid when the underlying stochastic process is a stationary Gaussian process. However, in many cases the assumption of Gaussian distribution may not be appropriate. For example, regions under separated flows that exhibit non-Gaussian features in the pressure fluctuations are characterized by high skewness and kurtosis.

To address the conditional simulation problem for non-Gaussian random fields, Elishakoff *et al.* [6] presented a method adopting the iterative procedure proposed by Yamazaki and Shinozuka [7] to generate samples of unconditional non-Gaussian fields. By constructing a mapping between Gaussian fields and non-Gaussian fields, the existing conditional simulation technique for Gaussian random fields is effectively employed for non-Gaussian random fields. The correlation distortion based transformation mapping error is utilized as a criterion for convergence of the iterative procedure. Hoshiya *et al.* [8] developed a theoretical formulation based on the conditional probability density function with the transformation of non-Gaussian random variables into Gaussian variables, considering examples of log-normal, exponential, Rayleigh, Gumbel and uniform distributions. Using the forward modified Hermite transform as a mapping scheme between Gaussian and non-Gaussian processes with desired values of skewness and kurtosis, Gurley and Kareem [9] introduced “spectral correction” method for conditional simulation of non-Gaussian processes to replace missing or damaged records. The simulation accurately maintained the correlation between multiple locations as well as the appropriate spectral and probabilistic contents of the processes at each location.

Despite the increasing attention the conditional simulation schemes have received in recent years, very limited studies have been devoted to the simulation of non-stationary fields. In general both the statistical moments and the frequency content of such non-stationary processes evolve in time, which makes conditional simulation of such random processes even more challenging. Heredia-Zavoni and Stanta-Cruz [10] mapped the non-stationary random fields to a domain where the conditional simulation is performed for stationary, space-time fields. They used a stochastic model for one dimensional earthquake ground motion [11] by analyzing evolutionary spectral density and cross-correlation structure of a class of non-stationary random fields in terms of envelop and frequency modulation functions. Morikawa and Kameda [12] introduced group delay time spectra to represent the properties of non-stationary processes in the numerical generation of conditional random fields containing non-stationary time series. These methods were straightforward extensions of those for the stationary processes based on Fourier coefficients/spectra.

In recognition of the recent developments in the time-frequency analysis tools, this study seeks to utilize time-frequency analysis tools to model the time-dependent characteristics involved in the conditional simulation of non-stationary random fields. Wavelet transform, a mathematical tool to represent non-stationary processes as a liner superposition of wavelet basis function has formed the basis of this study. An evolutionary cross-correlation structure of non-stationary random fields in terms of wavelet coefficients is constructed which effectively facilitates the extension of the Kriging method from stationary random to non-stationary random fields. A framework of generating interpolating stochastic processes for the conditional simulation of non-stationary space-time random fields is proposed. A numerical example related to the conditional simulation of gust-front wind velocities is presented to demonstrate the accuracy and efficacy of the proposed method.

2 THEORETICAL BACKGROUND

The problem of conditional non-stationary space-time random processes can be formulated as follows. Consider $U(x,t)$ as non-stationary space-time processes, whose several realizations $u(x_i,t)$ have been recorded at locations x_i $(i=1,2,...,n-1)$. The conditional processes $U(x_j,t\,|\,u(x_i,t))$ $(i=1,2,...,n-1;\ j=n,...,N)$ are to be estimated at some other locations of interest. The proposed iterative conditional simulation method employs collectively a Kriging method and Hermite and wavelet transforms, which are briefly described in the following.

2.1 *Kriging Method*

The Kriging estimate of unknown realization $u(x_n,t)$, denoted as $u(x_n,t)^e$, is interpolated linearly in terms of $n-1$ known realizations as follows:

$$u(x_n,t)^e = \sum_{i=1}^{n-1} \lambda_{in} u(x_i,t) \tag{1}$$

The Kriging weights λ_{in} can be determined using Lagrangian technique. By minimizing the estimation variance

$$E[(U(x_n,t)^e - U(x_n,t))^2] = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \lambda_{in}\lambda_{jn} R_{ij} - 2\sum_{i=1}^{n-1}\lambda_{in}R_{in} + R_{nn} \tag{2}$$

which subjects to the unbiased estimator condition

$$E[U(x_n,t)^e - U(x_n,t)] = E[\sum_{i=1}^{n-1} \lambda_{in} U(x_i,t) - U(x_n,t)] = \sum_{i=1}^{n-1} \lambda_{in}\mu_i - \mu = 0 \tag{3}$$

where E is the ensemble average; R_{ij} correlation function between U_i and U_j, λ_{in} is thus evaluated as follows

$$\begin{Bmatrix} \lambda_{1n} \\ \lambda_{2n} \\ \vdots \\ \lambda_{(n-1)n} \end{Bmatrix} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1(n-1)} \\ R_{12} & R_{22} & \cdots & R_{2(n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ R_{1(n-1)} & R_{2(n-1)} & \cdots & R_{(n-1)(n-1)} \end{bmatrix}^{-1} \begin{Bmatrix} R_{1n} \\ R_{2n} \\ \vdots \\ R_{(n-1)n} \end{Bmatrix} \tag{4}$$

At the same time, the errors $e(x_j,t)^s$ for a stochastic process with zero mean are unconditionally simulated using modal decomposition method on the basis of the covariance matrix (Hoshiya 1995) for the $N-n+1$ stochastic variates

$$V_{N-n+1} = \begin{bmatrix} E[e(x_n,t)^{s^2}] & E[e(x_n,t)^s e(x_{n+1},t)^s] & \cdots & E[e(x_n,t)^s e(x_N,t)^s] \\ E[e(x_n,t)^s e(x_{n+1},t)^s] & E[e(x_{n+1},t)^{s^2}] & \cdots & E[e(x_{n+1},t)^s e(x_N,t)^s] \\ \vdots & \vdots & \ddots & \vdots \\ E[e(x_n,t)^s e(x_N,t)^s] & E[e(x_{n+1},t)^s e(x_N,t)^s] & \cdots & E[e(x_N,t)^{s^2}] \end{bmatrix} \tag{5}$$

which is established according to the following relations:

$$E[e(x_n,t)^s] = E[(U(x_n,t) - U(x_n,t)^e)] = 0 \tag{6}$$

$$E[e(x_n,t)^s e(x_j,t)^s] = R_{nj} - \sum_{i=1}^{n-1}\lambda_{in}R_{ij} = R_{nj} - \sum_{i=1}^{n-1}\lambda_{ij}R_{in} \tag{7}$$

Finally, the conditionally simulated values are obtained as

$$u(x_j,t)^c = u(x_j,t)^e + e(x_j,t)^s \qquad (j = n,\ldots,N) \tag{8}$$

Due to the underlying assumption of $u(x_i,t)$ as a stationary Gaussian random process for the Kriging method, it is necessary to introduce modified Hermite transform and a time-frequency analysis tool, wavelet transform, to allow the extension of Kriging method to non-stationary/non-Gaussian random processes.

2.2 *Modified Hermite Transform*

The forward modified Hermite transform (Gurley & Kareem 1998) generates a non-Gaussian process x_{ng} through static transformation of a Gaussian process x_g

$$x_{ng} = x_g + d_3(x_g^2 - 1) + d_4(x_g^3 - 3x_g) \tag{9}$$

where the appropriate values of the polynomial coefficients d_3 and d_4 are tailored such that x_{ng} matches the target skewness and kurtosis. Correspondingly, the backward modified Hermite transform identifies the desired coefficients in the inverse relationship of Equation 9 necessary to produce a Gaussian process x_g from a non-Gaussian process x_{ng}. Accordingly, the forward and backward modified Hermite transform allows a convenient mapping between Gaussian processes and non-Gaussian processes and facilitates application of the Kriging method to non-Gaussian processes.

2.3 *Wavelet Transform*

The wavelet transform has been of particular interest for the analysis of signals characterized by transient behavior or discontinuities. Aiming to represent a signal as a linear superposition of basis functions, wavelet transforms are localized and dependent only on the local properties of a signal in the neighborhood [13]. As a convenient tool to extract time-frequency information from a non-stationary signal, wavelet transform has found a number of applications in engineering in recent years. Through a set of basis functions, i.e., the dilation and translation of the parent wavelet function, the wavelet transform provides a bank of wavelet coefficients representing a measure of similitude between the basis function and the signal at time t and scale a, as shown below:

$$W_x(a,t) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} X(t)\phi^*(\frac{t-\tau}{a})dt \tag{10}$$

Accordingly, for discrete values of time t and scale a, a discrete wavelet transform (DWT) is defined as:

$$W_x(i,j) = \sum_n X(n)\psi_{i,j}(n) \tag{11}$$

where $\psi_{i,j}(n)$ represents the wavelet basis function. The dilation of a discrete wavelet can be viewed as a tree of low and high pass filtering operations followed by a sub-sampling by 2 which successively decompose a signal akin to a dyadic filter bank.

Computationally, the DWT is very efficient. However, the classical DWT does not exhibit the desirable property of shift-invariance, i.e., in general the DWT of a translated signal is not the translated version of the DWT of the signal. To overcome this and provide more precise information for the frequency localization, the stationary wavelet transform (SWT) has been proposed as a special type of DWT (14). Excluding the step involving sub-sampling, SWT ensures the desirable property of translation invariance and requires more computations. This study utilizes SWT in the proposed simulation algorithm and invokes decomposition of a non-stationary process into a summation of mono-component processes:

$$X(t) = \sum_{n=1}^{N} D_n(t) + A(t) \tag{12}$$

in which N represents the level of decomposition; $D_n(t)$ denotes the detail function at level n and $A(t)$ is the approximation function, which represents the trend in $X(t)$. The SWT decomposition facilitates the definition of time-varying correlation map between the wavelet coefficients of processes $u(x_i,t)$ and $u(x_j,t)$ at each level of the decomposition. Since the localized wavelet coefficients $W_x(a,t)$ represent the energy at corresponding time intervals of the signal $x(t)$, the representation of the squared wavelet coefficients in the time-scale domain has been referred to as scalogram, which represents time-varying energy over frequency. Correspondingly, to identify correlation between signals, the squared coefficients are replaced with the average of the product of the wavelet coefficients of two different processes, which provides a view of the coincident events between the processes, revealing time-varying pockets of correlation over frequency. The wavelet-based correlation map may be expressed as follows

$$R_{ij}(a,t) = \int_T W^*_{u_i}(a,\tau) W_{u_j}(a,\tau) d\tau / N \tag{13}$$

in which the localized time integration widow $T = [t - N\Delta t/2, t + N\Delta t/2]$ is selected based on the time resolution desired in the resulting correlation map. It will be shown in the next section that the SWT allows convenient extension of traditional Kriging method to time-frequency domain for non-stationary random fields.

3 CONDITIONAL SIMULATION SCHEME

The effectiveness of wavelet transform to present transient characteristics of non-stationary processes has led to propose here the use of Kriging estimates of unknown realization $u(x_n,t)$, denoted as $u(x_n,t)^e$ in terms of stationary wavelet coefficients. Now the problem becomes interpolating wavelet coefficients of unknown realizations, $w_{u_n}(a,t)$, in terms of wavelet coefficients $w_{u_i}(a,t)$ $(i = 1,...,n-1)$ of realizations at $n-1$ locations. With the aid of the Kriging method, invocation of the modified Hermite transform and the wavelet transform the proposed iterative conditional simulation method proceeds as follows. First, apply SWT to $n-1$ known observations $u(x_i,t)$ recorded at locations x_i $(i = 1,2,...,n-1)$ and obtain the corresponding wavelet coefficients $w_{u_i}(a,t)$ $(i = 1,...,n-1)$. This is followed by finding time-varying correlation map between the wavelet coefficients of processes $u(x_i,t)$ and $u(x_j,t)$, i.e., $R^T_{ij}(a,t)$, using Equation 13, which is then designated as design correlation $R^D_{ij}(a,t)$. The envelop functions $E_{u_i}(a,t)$ are determined by spline fitting, which is utilized to demodulate wavelet coefficients and determine the correlation at each level as $\hat{w}_{u_i}(a)$ and $R_{ij}(a)$. The first iteration is started. With the aid of forward and backward modified Hermite transformations, the conditional simulation is carried out using the Kriging method for Gaussian processes. Like the conventional Kriging simulation, the best linear unbiased estimate (BLUE) of $U(x_j,t \mid u(x_i,t))$ $(i = 1,2,...,n-1;\ j = n,...,N)$ is determined using Equations 1 and 4. Unconditional errors $e(x_j,t)^s$ based on the covariance matrix described in Equation 5 are obtained using the Cholesky or modal decomposition method. Thus the conditionally simulated wavelet coefficients for unknown realizations are obtained (Eq. 8). The resulting wavelet coefficients $\hat{w}_{G,u_m}(a)$ for location x_m are then transformed back to non-Gaussian distribution $\hat{w}_{NG,u_m}(a)$. By introducing envelop functions, simulated wavelet coefficients become $w_{u_m}(a,t)$ with correlation structure $R^E_{ij}(a,t)$. The resulting correlations are then compared to the target values and error is defined as

$$err(a,t) = \left| \frac{R^T_{ij}(a,t) - R^E_{ij}(a,t)}{R^T_{ij}(a,t)} \right| \tag{14}$$

If the error is unacceptable, second iteration is invoked by sending corrected design correlation back to the beginning of the simulation loop which results in the revised value of $\hat{w}_{u_m}(a)$. The iterations continue, until the correlation converges to the target correlation within the prescribed tolerance. In this manner, the time-dependent correlation of the resulting simulated wavelet coefficients matches the target within user-specified tolerance. In the end, the simulation is completed by the inverse stationary transform of simulated wavelet coefficients $w_{u_i}(a,t)$, that is, reconstruction based on the multilevel stationary wavelet decomposition structure. The schematic of the simulation procedure is demonstrated in Figure 1.

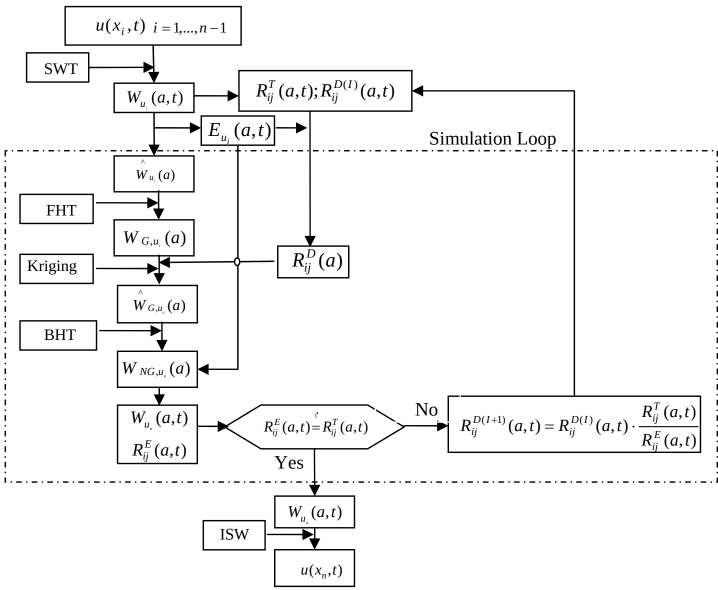

Figure 1: Schematic of the conditional simulation method

Comparison of this procedure with the conventional Kriging method reveals that the difference lies in the way the correlation map is presented. With the aid of the wavelet transform, time localized cross correlation of two signals is captured and the Kriging coefficients are weighted on the time and frequency localized wavelet coefficients. However, with the use of Fourier transform involved in conventional Kriging method, the underlying global decomposition makes it unable to faithfully preserve and demonstrate the inherent non-stationary characteristics. In addition to the mathematical elegance, the proposed simulation method based on the Kriging method and SWT is computationally efficient, as will be shown in the following numerical example.

4 EXAMPLE

To demonstrate the effectiveness and efficacy of the proposed simulation method, a set of downburst wind data is employed herein. The rapid variation in wind speed and direction associated with thunderstorm downbursts characterize wind time histories as non-stationary processes with time-varying mean, variance and frequency contents. Thunderstorm related winds, e.g., gust fronts are ideal for the conditional simulation of non-stationary processes as often very limited data is gathered and there is always a need for data at additional locations, therefore, it has been chosen for this example.

The data is simultaneously recorded on June 15, 2002 at different heights in a field experiment by the Department of Atmospheric Science at Texas Tech University (15). Following the proposed iterative conditional simulation procedure, the wind speeds at 10m height x_3 is simulated based on the data measured at 4m (x_1) and 15m (x_2) heights (as shown in Fig. 2). The simulation preserves the temporal fluctuations of the real measurement at 10m height, as compared in Figure 3. The time-varying correlation structure, including the simulated and target $R_{33}(a,t)$, $R_{13}(a,t)$ and $R_{23}(a,t)$ are compared in Figures 4-6, in which red lines

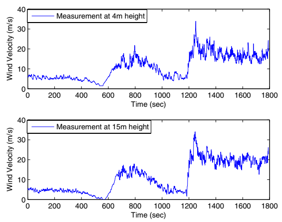

Figure 2: Measurements

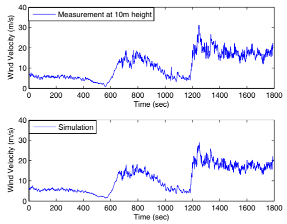

Figure 3: Comparison of Measurement and Simulation

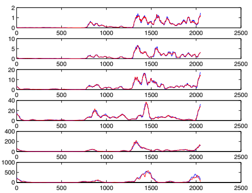

Figure 4: $R_{33}(a,t)$

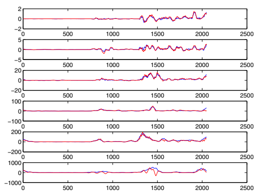

Figure 5: $R_{13}(a,t)$

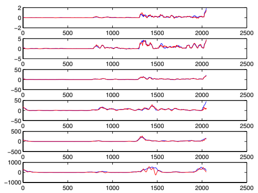

Figure 6: $R_{23}(a,t)$

represent simulation results and blue lines represent target. The simulated results accurately maintain the correlation between multiple locations.

5 CONCLUDING REMARKS

A framework for the conditional simulation of non-stationary/non-Gaussian space-time random processes with target correlation structure is proposed. The simulation framework utilizes wavelet transform and modified Hermite transform as a format for time-frequency modeling of non-stationary processes and a mapping between Gaussian and non-Gaussian processes, respectively in the application of the Kriging method to simulate non-stationary/non-Gaussian processes. The introduction of wavelet coefficients based correlation help extend Kriging method from time/frequency domain to time-frequency domain. The focus of the simulation scheme on non-stationary and non-Gaussian random processes provides a new perspective for the conditional simulation of these processes using time-frequency framework. Applications of this scheme to transient wind events are particularly noteworthy as on very limited occasions such events are captured at limited locations, which can be conveniently expanded by the proposed scheme.

6 ACKNOWLEDGEMENTS

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