RELIABILITY OF RANDOM DECREMENT TECHNIQUE FOR ESTIMATES OF STRUCTURAL DAMPING

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Abstract

The following study investigates the reliability of Random Decrement Signatures in the presence of non-stationarity and when correlation is allowed between samples averaged in the signature. The effect of such features upon signature variance was not investigated previously and was facilitated through the use of bootstrapping theory. A limiting case error is proposed in light of the findings of this study to permit variance estimates when correlations between samples are permitted, greatly increasing the number of samples available for averaging. In general, the effects of correlation are marginal, while the two classes of non-stationarity considered produce significant deviations in error estimates from theory.

Introduction

The Random Decrement Technique (RDT) has become one of the most popular approaches used for estimation of structural stiffness and damping from wind-excited responses, because of its ability to overcome the strict requirements for lengthy stationary data imposed by traditional spectral approaches, for which the narrowband response of typical structures would necessitate perhaps a hundred or more hours of data to minimize bias and variance errors. Its widespread use served as a motivation for the current study to explore some of its limitations, specifically, the reliability of these decrement signatures when idealized assumptions are no longer valid. Previous authors (Kareem and Gurley, 1996; Spanos and Zeldin, 1998) provided some discussion of the implications of the white noise excitation assumption, illustrating that the Random Decrement Signatures (RDS) cannot be equal to the free vibration curve if the excitation is not truly white. However, there has been no treatment of the implications of stationarity assumptions and correlation between samples. Yet these conditions may be commonly encountered in practice. Thus, the following study examines these issues and provides a simple measure of reliability using bootstrapping theory to estimate the variance of RDS.

The decrement is generated by capturing a prescribed length of the time history upon the satisfaction of a threshold condition (Cole, 1973). This triggering condition, in its strictest sense, will specify both amplitude and slope criteria. The segments meeting these conditions are averaged to remove the random component of the response, assumed to be zero mean, leaving the autocorrelation signature ($R_X(t)$) for the system. Thus, the RDS is expressed as an expectation shown (Vandiver *et al.*, 1982) to reduce to

$$D_{X_p}(t) \equiv E[X(t_2) | X(t_1) = X_p \;\&\; \dot{X}(t_1) = \dot{X}_p] = X_p R_X(t) / R_X(0) \qquad (1)$$

The method by which segments are captured varies in the literature, with the threshold condition relaxed at times to merely specify a sign to the slope at that point; however, strict adherence to the condition may only be achieved by defining a specific value to the triggering slope, e.g. by capturing only peaks, for which the slope is zero. This strategy was proposed by Tamura and Suganuma (1996) to permit more precise amplitude-dependent damping estimation and shall be used in this study as a strict triggering condition. Unfortunately, this condition will require more data, thus any peak within a percentage (e.g. 3%) of X_p may be retained to generate more candidate samples. Irregardless, the RDS variance can be expressed in closed form (Vandiver *et al.*, 1982):

$$Var[D_{x_o}(t)] = E[D^2{}_{x_o}(t)] - E[D_{x_o}(t)]^2 = R_x(0)/N[1 - R_x^2(t)/R_x^2(0)] \qquad (2)$$

where N is the number of averages in the estimate. Note that the variance estimated here was derived using the assumptions of a linear oscillator excited by Gaussian, zero mean white noise. Under these conditions, the autocorrelation, normalized by C, has the form:

$$R_x(t) = Ce^{-xwt}\cos(w_D t) \qquad (3)$$

which is analogous to the free vibration of a system with critical damping ratio of x, natural frequency of w, and damped natural frequency of w_D. It was further assumed that averaged segments do not overlap one another in time, therefore allowing no correlation. The implications of this assumption are further discussed in a subsequent section.

Bootstrapping Estimates of Variance

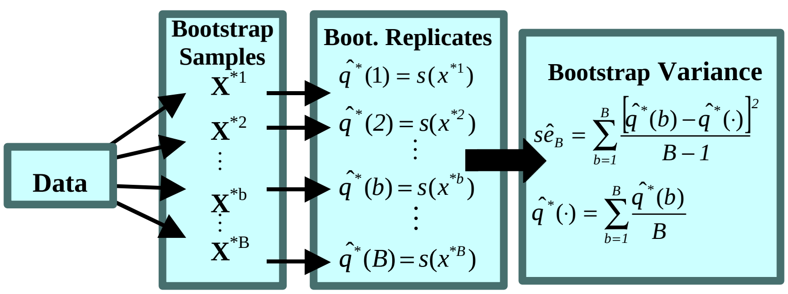

Figure 1. Schematic representation of bootstrapping notion.

In practice, any one of the above assumptions may be violated. Thus the closed-form variance of (2) may not be accurate. At the same time, calculating an estimate of variance directly from the N segments of the RDS yields merely a sample variance. Thus, throughout this paper, the notion of bootstrapping will be exercised as an alternative means by which to estimate the true variance of the signatures (Efron and Tibshirani, 1993). In this approach, at each time step, bootstrap samples of length N are generated by randomly sampling, with replacement, from the measured data. These are averaged to form a bootstrap replicate. The process, shown in Figure 1, is repeated B times, and these replicates are used to estimate the variance of the data. In particular, the bootstrapping technique will be used in this study not only to provide an estimate of the variance in the RDS at every point in time, but the replicate signatures generated through this process will be valuable in defining confidence envelopes in the subsequent examples (see Figure 2). It is hoped that the introduction of such a scheme will provide practitioners with a simple means by which to estimate the variance of their RDS and provide a measure of the reliability when theoretical assumptions are not entirely met.

Parameter Study

The sensitivity of RDS to the violation of various assumptions in its theory will be explored in the subsequent sections. The analyses were conducted on a SDOF oscillator excited by Gaussian, zero-mean, white noise, unless stated otherwise. The response of the 0.2 Hz system with 1% critical damping was simulated for 12 hours, sampled at 2 Hz. All bootstrapped estimates are based on *B=50* replicates.

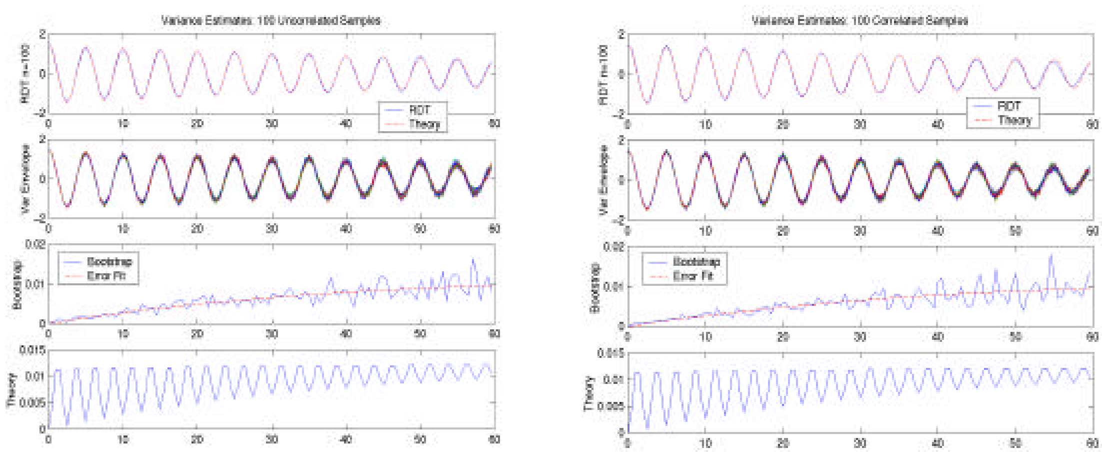

Figure 2. (top to bottom) RDT against theoretical autocorrelation; bootstrap variance envelope; bootstrapped variance estimate against (4); theoretical variance (2); Signatures with correlation prohibited (left) and correlation permitted.

Suggested Expression for Random Decrement Signature Variance

The examination of simulated data as shown in Figure 2 agrees in trend with the theoretical expression given in (2). As the initial conditions are indeed enforced at *t=0*, the RDS is theoretically most reliable at this point. Subsequently, the signature increasingly breaks down further from this point. The bootstrapped replicates of the decrement signature shown as the second of a sequence of plots in Figure 2 yields some appreciation of the variance in estimates, propagating with time. Note also that the closed form expression for variance in (2) is based upon the autocorrelation function in (3) and yields results that are counter-intuitive to the actual behavior. In theory, the variance will oscillate as the modulus of a cosine function, indicating that it will reach maximum every half cycle. With time, these oscillations diminish and the variance will approach a near constant value of the signal variance divided by *N*. This equation would also indicate that within just a few seconds of the trigger threshold, the variance has already reached its maxima, contradicting the bootstrapped estimates and intuitive arguments presented in Vandiver *et al.* (1982). A more practical, limiting case for the error may be given by:

$$e = R_X(0)/N\left(1-e^{-2xwt}\right) \tag{4}$$

As shown by Figure 2, this limiting error fits well against the fluctuating variance of the RDS for both the non-correlated and correlated cases. For the latter case, there is no closed form. This expression will be used throughout the paper for comparison.

Required Number of Ensembles

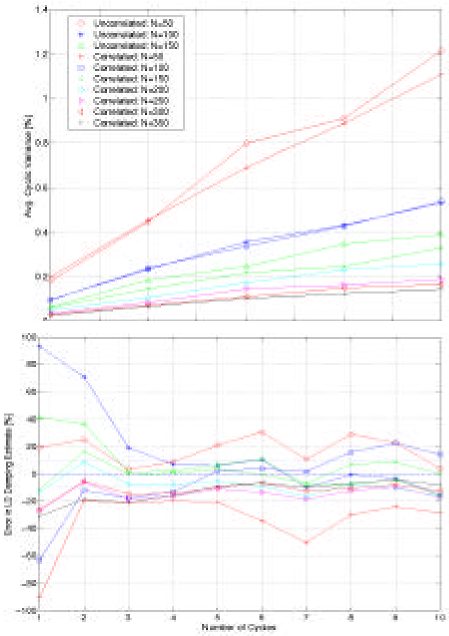

Figure 3. RDT Cyclic variance (top) and damping estimation error.

Despite the widespread use of the RDT, there still is much debate surrounding the amount of data required to yield reliable estimates. As shown by Figure 3, by averaging the bootstrap variances over each cycle of the RDT signature, one can monitor the increase in variance as more cycles are considered in the estimate. Clearly, as the number of segments averaged (*N*) increases, this variance decreases. However, it is interesting to note whether or not variance is a good indication of the accuracy of a given damping estimate. The error of logarithmic decrement estimates averaged over a number of cycles is also shown in Figure 3. The reliability of single cycle damping estimates is poor even though this is where the variance in the RDT estimate is minimum. Thus, a more accurate means of damping estimation is required to permit estimates using only first cycle data, which is more reliable. In fact, only by considering 3 or more cycles do estimates approach an accuracy of 10%; however, there is a trade-off in that the variance in the estimates also increases with the number of cycles. Thus the estimates of damping are best when performed over the first 5 cycles with more than *N*=200 averages.

Effects of Correlated Samples

The closed form expressions for the variance in (2) were based upon the assumption that segments averaged in the RDT were uncorrelated. Researchers have expressed the need for additional work to establish the impacts of correlation. As correlation is often permitted in practice, it was of interest to investigate the implications of the violation of this assumption through some simulations, as theoretical developments can provide no insight into the ramifications. According to Figure 2, one can see that the effects of correlation on the quality of the estimates are not considerable. In fact, aside from some random fluctuations in the bootstrapped variance estimate, the correlated case produces the same limiting variance as the uncorrelated case, fit by (4). By examining Figure 3, one can see that here in no increase in cyclic variance as a result of permitting some correlation between samples. In fact, the examples illustrate that there is actually a marginal decrease in the variance for the same number of averages when correlation is allowed. By allowing some overlap between adjacent samples, the practicing engineer is now afforded additional samples for averaging, a critical requirement for the use of RDT.

Limitations of Stationary Assumptions

As the requirements of lengthy stationary data records had often precluded the use of traditional spectral and autocorrelation techniques, the RDT was proposed as a way to circumvent this problem by permitting analysis on shorter lengths of stationary data. Though it is often assumed that wind-induced response of structures is stationary, examination of full-scale data has often proven otherwise. Even so, there has been little treatment of the ability of RDT to perform under non-stationary conditions (Jeary, 1992). To illustrate the implications of stationarity on the RDT, two cases were studied using the same oscillator. In case 1, the random input is comprised of two, 4 hour blocks of standard Gaussian white noise (segments 1 & 3) separated by a 4 hour block of zero mean, white noise drawn from a uniform distribution (segment 2). The definitions of stationarity require that all statistical properties be invariant with time, strictly implying that all the data be drawn from the same distribution. A sudden pocket of non-stationarity is introduced to violate this assumption.

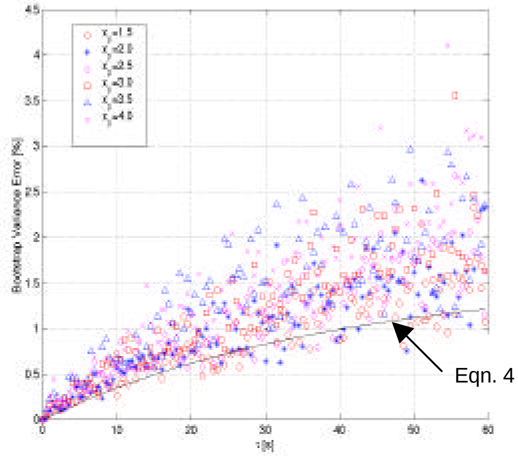

Figure 5. Bootstrap variance estimates for non-stationary case 2.

Figure 4 illustrates the implications of the violation of stationarity in case 1 by examining the bootstrapped estimate of variance for several triggering thresholds. Note that in each of these cases, the same number of segments (*N=200*) was averaged. Although the variance is theoretically independent of triggering level, as evidenced by (2), this figure displays an increase in variance with triggering level when compared to the limiting variance function of (4). These findings may be rationalized by in light of the histograms of the peaks within each block of data, which were omitted for brevity. Over ninety percent of the high amplitude peaks are located in segments 1 & 3. As a result, when using these higher amplitude trigger conditions, shown in blue, there may be only a few isolated samples drawn from segment 2. The Gaussian samples by themselves are incapable of averaging out the variance of the isolated samples from segment 2, thus leading to corrupted results for these higher trigger levels. Conversely, low amplitude trigger conditions shown in shades of red have a sufficient number of samples drawn from segment 2 to cancel out the variance from the uniformly distributed response component. In light of this, one may suggest the use of only low trigger levels; however, if the RDT is being used to investigate some amplitude referencing, as suggested by

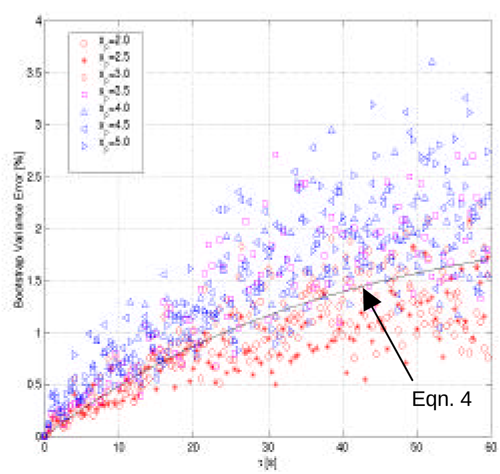

Figure 4. Bootstrap variance estimates for non-stationary case 1.

Jeary (1992) and Tamura and Suganuma (1996), then higher amplitude triggering conditions must also be considered. In such cases, RDS may appear accurate but a closer look reveals higher levels of variance that result from isolated non-stationary pockets.

A second case of non-stationarity was investigated by enveloping the excitation by a sinusoidal function. As opposed to the previous instance, this case will consider a global phenomenon, with a sinusoid of 2-hour period modulating the Gaussian, white noise excitation. As shown by Figure 5, in this case the dependence upon triggering amplitude is of course not present, as all levels of triggering reflect the same poor performance when compared to variance estimate in (4), although higher trigger levels seem to scatter toward higher variances. This reflects the power of global non-stationary features in the data, as the variance of estimates will be far greater than idealized theory would predict.

Conclusions

The present study investigates the effects of non-stationarity and correlation upon RDS. As the closed form expressions for variance are not applicable when ideal assumptions are violated, bootstrapping theory is introduced as an alternative means to estimate variance. The effects of correlation upon RDS were found to be marginal, with variance matching well with a proposed formula. The study further examined the effects of non-stationarity and noted a significant increase in variance in comparison to idealized theory. As nothing is truly stationary in practice, the bootstrapped variance estimates may become a valuable tool to establish the reliability of Random Decrement estimates.

Acknowledgements

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