Model Predictive Control For Wind Excited Buildings: A Benchmark Problem

Gang Mei, Student M. ASCE, Ahsan Kareem, M. ASCE, and Jeffrey. C. Kantor
University of Notre Dame, Notre Dame, IN 46556, USA
gmei@nd.edu, kareem@nd.edu, and Kantor.1@nd.edu

ABSTRACT

In this paper, a third generation benchmark problem for response control of wind excited tall buildings defined by Yang, et al. (1999) is studied by using the Model Predictive Control (MPC) scheme. A 76 story, 306 meters concrete office tower proposed for the city of Melbourne, Australia, is being used to demonstrate the control scheme. The MPC scheme is based on an explicit use of a prediction model of the system response to obtain the control action by minimizing an objective function. Optimization objectives in MPC include minimization of the difference between the predicted and desired response trajectories, and the control effort subjected to certain constraints. MPC considers input/output hard constraints and provides an optimal control force within prescribed limitations.

Keyword: Benchmark, Wind-Excited, Model Predictive Control, Constraints

1. INTRODUCTION

Structural control is an attractive option for improving the performance of a variety of structures, including bridges, tall buildings, and offshore structures under earthquakes and strong winds. A wide range of control devices and schemes have been proposed and implemented in the area of structural control (Soong, 1990; Houser et al., 1997; Kareem and Kijewski, 1999). To compare the different control schemes, Yang et al. (1998) proposed the second generation benchmark problem for wind-excited building in the Second World Conference on Structural Control. These benchmark problems are ideally suited for comparing the performance of different control schemes and devices. Now, the third generation benchmark problem of wind-excited buildings has been proposed (Yang et al., 1999). In this study the wind load time history was obtained from a wind tunnel test in Sydney University for use in the time domain analysis.

In this paper, a reduced order model for the 76 story concrete building is controlled by using MPC. MPC provides an alternative simple control method and can handle constrained problems conveniently (Mei et al., 1998). Two cases are considered here. One is MPC only, which does not include the constraints in objective function. The constraints are satisfied by choosing weighting matrices. The other case is MPC constrained case which searches the constrained space for an optimal solution. The inequality constraints on the maximum control force and AMD displacement are included in optimal objective. At each time step MPC reduces to an optimization problem subject to inequality constraints. A quadratic programming algorithm is used to obtain the optimal control force (Mei et al., 2000).

2. PROBLEM DESCRIPTION

The benchmark problem in Yang et al. (1999) involves a 76 story, 306 meters concrete office tower subjected to the across wind excitation. An active mass damper was installed on the top floor. An evaluation model which was obtained by model reduction has 48 states. These procedures simplified the computation efforts. The equations of motion were expressed in a state space form:

$$\dot{x} = Ax + Bu + EW \text{ , } z = C_z x + D_z u + F_z W \text{ , } y = C_y x + D_y u + F_y W + v \tag{1}$$

where $x = [\bar{x}, \dot{\bar{x}}]^T$ is the 48-dimensional state vector; u is the scalar control force; W is the wind excitation vector of dimension 24; $z = [\bar{z}, \dot{\bar{z}}, \ddot{\bar{z}}]^T$ and $y = [\dot{\bar{z}}, \ddot{\bar{z}}]^T$ are control output vector and measured output vector of the evaluation model; v is a vector of measured noise; x_m is the relative displacement of the mass damper with respect to the top floor. The definitions of these variables and matrices A, B, E, C_z, D_z, F_z, C_y, D_y and F_y were given by Yang et al (1999) and have appropriate dimensions.

The wind force data acting on the benchmark building were determined from wind tunnel tests. For the performance evaluation of control systems, the first 900 seconds of across wind data were used for computation of building response. Time domain analysis was conducted on this evaluation model. Twelve evaluation criteria were defined about rms values and peak response values of displacement, acceleration at different floor, and controller performance. This information can be obtained from the paper by Yang et al. (1999).

3. MODEL PREDICTIVE CONTROL SCHEME

The MPC scheme is based on an explicit use of a prediction model of the system response to obtain the control action by minimizing an objective function. The optimization objectives include minimization of the difference between the predicted and desired response and the control effort subject to certain constraints such as limits on the magnitude of the control force. In MPC scheme, first a reference trajectory, $y_r(k)$, is specified. The reference trajectory is the desired target trajectory for the process output. This is followed by an appropriate prediction model which is then used to determine the future building responses, $\hat{y}(k)$. The prediction is made over a preestablished extended time horizon using the current time as the prediction origin. For a discrete time model, this means predicting $\hat{y}(k+1)$, $\hat{y}(k+2)$, …, $\hat{y}(k+i)$ for i sample times in the future. This prediction is based on both actual past control inputs $u(k)$, $u(k-1)$, …, $u(k-j)$ and on the sequence of future control efforts determined using the prediction model that are needed to satisfy a prescribed optimization objective. The control signals that were determined using the prediction model are then applied to the structure, and the actual system output, $y(k)$, is found. Finally, the actual measurement, $y(k)$, is compared to the model prediction $\hat{y}(k)$ and the prediction error ($\hat{e}(k) = y(k) - \hat{y}(k)$) is utilized to update future predictions.

In the general model predictive control, the discrete-time state-space equations of the system are used and to estimate the future state of the system:

$$\begin{aligned}\hat{x}(k+1|k) &= \Phi\hat{x}(k|k-1) + \Gamma_u\hat{u}(k|k-1) + \Gamma_e\hat{e}(k|k)\\ \hat{z}(k|k-1) &= C_z\hat{x}(k|k-1) + D_z\hat{u}(k|k-1)\\ \hat{y}(k|k-1) &= C_y\hat{x}(k|k-1) + D_y\hat{u}(k|k-1)\end{aligned} \tag{2}$$

where $\hat{x}(k+1|k)$ is the estimator of the states at future sampling period $k+1$ based on the information available at period k, $\hat{y}(k|k-1)$ is the estimator of the plant output at period k

based on information at period $k-1$, Γ_e is the Kalman-Bucy estimator gain matrix and $\hat{e}(k|k)$ is the estimated error: $\hat{e}(k|k) = y(k) - \hat{y}(k|k-1)$

By using Eq. 2, the process output predicted at the kth step and the subsequent time steps $k+j$, $j = 1, \ldots, p$ can be expressed as a function of the current state vector $x(k)$ and control vector $\mathbf{u}(k) = \left[\hat{u}^T(k+1|k) \ \ldots \ \hat{u}^T(k+\lambda-1|k)\right]^T$ as follows:

$$\Psi(k) = H\mathbf{u}(k-1) + Y_z\hat{x}(k|k-1) + Y_e\hat{e}(k|k) \tag{3}$$

and $\Psi(k) = \left[\hat{y}^T(k+1|k) \ \ldots \ \hat{y}^T(k+p|k)\right]^T$. p is the prediction horizon and λ is the control horizon. Therefore, the objective function is given by

$$J = \frac{1}{2}\Psi(k)^T\bar{Q}\Psi(k) + \frac{1}{2}\Delta\mathbf{u}^T(k)\bar{R}\Delta\mathbf{u}(k) \tag{4}$$

subject to the linear inequality constraints:

$$\mathbf{u}(k) \geq \mathbf{u}_{min}(k),\ \mathbf{u}(k) \leq \mathbf{u}_{max}(k),\ \Psi(k) \geq \Psi_{min}(k),\ \Psi(k) \leq \Psi_{max}(k) \tag{5}$$

To solve the problem, a quadratic programming algorithm is used. Using $\nu(k) = \mathbf{u}(k) - \mathbf{u}_{min}(k)$, the optimization problem can be written as a standard quadratic programming problem which uses an active set algorithm to obtain the optimal solution. An iterative sequence of feasible points is generated that converges to the solution. The optimal predictive control force is obtained by an optimal value in the constraint set which minimizes the performance function (Mei et al, 1999). In the case of no constraints, the control force can be explicitly written as:

$$\mathbf{u}=[H^T\bar{Q}H + \bar{R}]^{-1}H^T\bar{Q}[Y_z\hat{x}(k|k-1) + Y_e\hat{e}(k|k) - R_d\bar{R}u(k-1)] \tag{6}$$

in which H, $\bar{Q}$, $\bar{R}$, R_d, Y_z, Y_d and Y_e were given in Mei et al. (1998).

4. APPLICATIONS

In this study, Kalman-Bucy filter is used to obtain the feedback gain of the observer. Three accelerometers are used to measure the accelerations and estimate the states of the structure.

4.1 Nominal Building

First the nominal building with designed stiffness is studied using the MPC scheme without considering a hard constraint. The limit on the control force and AMD’s displacement are satisfied by adjusting weighting matrices Q and R. Then MPC considering constraints on the control force and displacement of AMD is applied for this nominal building. Table 1 gives performance criteria under across wind excitation using MPC with no constraints (MPC^1) and MPC with constraints (MPC^2), respectively.

Table 2 shows the peak and rms values of displacement, acceleration at different floors. For the MPC without considering the constraints, as mentioned by Mei et al. (1998), MPC has control effectiveness equivalent to the LQG control scheme. Using the MPC scheme the peak and RMS values of structural response are at same level as those of LQG.

Then the controlled response using MPC with constraints is evaluated. The constraints on the control force are set $\left[-120KN \ 120KN\right]$ as an example. The constraint on the output is limitation of the displacement of the AMD which requires that the maximum displacement be less than 95 cm. The control force reaches the constraint lines and an optimal solution within the boundary is obtained by MPC's constraint scheme.

4.2 Buildings with ±15% of Original Stiffness

To show the robustness of the controller, the uncertainty of building stiffness is taken into consideration. In addition to the "nominal building" above, two additional buildings are considered. One with +15% higher stiffness matrix and the other with a -15% lower stiffness. The controller obtained previously for the nominal building is applied to the ±15% buildings. The performance criteria of the ±15% buildings under across wind load are presented in Table 3. The RMS and peak values of displacement and acceleration of the two building are listed in Table 4 and 5.

As shown in these tables, MPC^1 designed for the nominal building reduces response of the +15% buildings. However, it gives less reduction in displacement and acceleration than LQG. For MPC^2 scheme, the maximum control force is set at 120 KN. The control performance is better than those of MPC^1 and LQG. The RMS value of the AMD's displacement is 34.45 cm because AMD approaches displacement boundary more often in the constrained case. The peak value of AMD's displacement is at 93.3cm.

To sum up, from the numerical examples, MPC exhibits effectiveness similar to the LQG method. The ±15% changes in the stiffness of building does affect the controller performance greatly. MPC shows good robustness when there exists uncertainty in the structural model. MPC can also deal with the structure control under constraints effectively. Simulations show that for the AMD, MPC with constraints can restrict the control force within the limits and generate optimal control force at each time step. The damper displacement is also limited within a certain range. Above all, the MPC scheme simulates practical problems and provides a good way to handle the constraints that were ignored in the previous works.

5. CONCLUSION

In this paper, the MPC scheme was employed to reduce structural response of the benchmark problem under wind excitation when the structure and control device are subjected to inequality constraints. An optimal solution was found within prescribed limits for controller design. Numerical results demonstrated the effectiveness of the MPC scheme with or without the consideration of constraints. The MPC only case is comparable with the LQG method and has the same control effectiveness. The constraints on the AMD can be handled by the MPC constrained scheme. The method provides a reliable and computationally convenient way to study and design devices for structural control under constraints.

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APPENDIX

Table 1: Evaluation criteria for across-wind excitations

RMS response (ΔK=0%)			Peak Response (ΔK=0%)		
Eval. Criteria	MPC^1	MPC^2	Eval. Criteria	MPC^1	MPC^2
J_1	0.367	0.318	J_7	0.401	0.369
J_2	0.415	0.362	J_8	0.452	0.441
J_3	0.575	0.548	J_9	0.723	0.714
J_4	0.576	0.550	J_{10}	0.732	0.723
J_5	2.325	2.835	J_{11}	2.296	2.671
J_6	13.69	23.05	J_{12}	84.27	103.56
σ_u (kN)	34.86	50.34	$max\|u(t)\|$	129.9	120
σ_{xm} cm	23.57	28.73	$max\|x_m\|$	74.17	86.26

Table 2: Results of MPC for nominal building

	MPC^1 u_{max} = 129.9 kN		MPC^2 u_{max} = 120.0 kN		MPC^1 σ_u = 34.86 kN		MPC^2 σ_u = 50.34 kN	
Floor No.	x_{pio} cm	$\ddot{x}_{pio}$ cm/s^2	x_{pio} cm	$\ddot{x}_{pio}$ cm/s^2	σ_{xi} cm	$\sigma_{\ddot{x}i}$ cm/s^2	σ_{xi} cm	$\sigma_{\ddot{x}i}$ cm/s^2
1	0.041	0.252	0.041	0.270	0.001	0.058	0.001	0.060
30	5.200	3.338	5.132	3.262	1.253	0.900	1.198	0.788
50	12.339	7.038	12.180	6.813	3.021	2.016	2.884	1.786
55	14.358	8.316	14.172	8.348	3.529	2.393	3.368	2.149
60	16.426	9.165	16.213	9.313	4.053	2.786	3.867	2.535
65	18.535	10.661	18.294	10.580	4.590	3.139	4.378	2.831
70	20.664	11.267	20.395	11.202	5.134	3.339	4.897	2.909
75	22.866	12.149	22.566	10.571	5.698	3.359	5.433	2.543
76	23.359	10.787	23.053	17.093	5.824	2.749	5.553	4.704
md	74.169	79.583	86.26	95.207	23.568	25.368	28.735	29.182

Table 3: Evaluation criteria for across-wind excitations

RMS response					Peak Response				
Eval. Criteria	ΔK=+15%		ΔK=-15%		Eval. Criteria	ΔK=+15%		ΔK=+15%	
	MPC^1	MPC^2	MPC^1	MPC^2		MPC^1	MPC^2	MPC^1	MPC^2
J_1	0.386	0.321	0.421	0.373	J_7	0.464	0.375	0.487	0.433
J_2	0.431	0.365	0.474	0.424	J_8	0.497	0.459	0.554	0.511
J_3	0.489	0.464	0.740	0.710	J_9	0.644	0.635	0.800	0.788
J_4	0.491	0.466	0.741	0.712	J_{10}	0.651	0.642	0.800	0.793
J_5	2.011	2.453	2.640	3.398	J_{11}	1.942	2.435	2.736	2.888
J_6	13.54	21.23	16.51	28.62	J_{12}	95.70	99.83	101.54	105.48
σ_u (kN)	42.33	53.93	38.57	58.85	$max\|u(t)\|$	152.8	120.0	136.26	120
σ_{xm} cm	20.38	24.87	26.76	34.45	$max\|x_m\|$	62.71	78.65	88.37	93.27

Table 4: Results of MPC for +15% building

	MPC^1 $u_{max} = 152.8$ kN		MPC^2 $u_{max} = 120.0$ kN		MPC^1 $\sigma_u = 42.33$ kN		MPC^2 $\sigma_u = 53.93$ kN	
Floor No.	x_{pio} cm	$\ddot{x}_{pio}$ cm/s^2	x_{pio} cm	$\ddot{x}_{pio}$ cm/s^2	σ_{xi} cm	$\sigma_{\ddot{x}i}$ cm/s^2	σ_{xi} cm	$\sigma_{\ddot{x}i}$ cm/s^2
1	0.037	0.273	0.036	0.272	0.008	0.064	0.008	0.063
30	4.612	4.984	4.540	4.076	1.071	1.288	1.018	0.940
50	10.952	7.829	10.811	7.332	2.577	2.150	2.447	1.827
55	12.749	8.791	12.589	8.492	3.009	2.451	2.857	2.154
60	14.593	9.768	14.411	9.680	3.454	2.825	3.278	2.527
65	16.475	11.629	16.269	10.957	3.910	3.224	3.710	2.832
70	18.377	12.609	18.144	11.382	4.373	3.528	4.148	2.938
75	20.344	14.062	20.082	11.291	4.851	3.527	4.600	2.572
76	20.784	20.795	20.516	17.591	4.958	5.375	4.702	4.940
md	62.707	82.060	78.655	96.357	20.383	22.920	24.87	28.291

Table 5: Results of MPC for -15% building

	MPC^1 $u_{max} = 136.3$ kN		MPC^2 $u_{max} = 120.0$ kN		MPC^1 $\sigma_u = 38.57$ kN		MPC^2 $\sigma_u = 58.85$ kN	
Floor No.	x_{pio} cm	$\ddot{x}_{pio}$ cm/s^2	x_{pio} cm	$\ddot{x}_{pio}$ cm/s^2	σ_{xi} cm	$\sigma_{\ddot{x}i}$ cm/s^2	σ_{xi} cm	$\sigma_{\ddot{x}i}$ cm/s^2
1	0.044	0.225	0.044	0.246	0.013	0.058	0.012	0.061
30	5.617	3.711	5.602	3.406	1.605	0.991	1.543	0.885
50	13.361	8.217	13.310	7.595	3.878	2.299	3.724	2.073
55	15.563	10.272	15.496	9.673	4.533	2.734	4.352	2.500
60	17.826	11.488	17.740	10.923	5.210	3.185	5.001	2.951
65	20.226	12.970	20.035	12.130	5.904	3.586	5.667	3.292
70	22.694	14.327	22.386	13.122	6.610	3.847	6.343	3.413
75	25.253	14.783	24.885	12.689	7.340	3.837	7.044	3.083
76	25.826	19.460	25.444	19.059	7.504	5.322	7.201	5.371
md	88.368	85.558	93.268	88.965	26.764	25.563	34.447	31.478

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